CHAPTER 1

Limits and Continuity

RADICACL EXPRESSIONS

Rationalizing the denominator

$$\frac{a}{b\sqrt{c}} = \frac{a}{b\sqrt{c}}\cdot\frac{\sqrt{c}}{\sqrt{c}} = \frac{a\sqrt{c}}{bc}$$

Conjugate Radicals

$$a+\sqrt{b} \leftrightarrow a-\sqrt{b}$$
$$\sqrt{a}+\sqrt{b} \leftrightarrow \sqrt{a}-\sqrt{b}$$
$$a\sqrt{b} \pm c\sqrt{d} \leftrightarrow a\sqrt{b} \mp c\sqrt{d}$$

PROPERTIES OF RADICALS

A. $a^{\frac{1}{n}} = \sqrt[n]{a}$

B. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

C. $\sqrt[n]{a^n} = a^{\frac{n}{n}} = a^1 = a$

D. $\sqrt[n]{ab} = \sqrt[n]{a}\cdot\sqrt[n]{b}$

Rationalize the denominator $\frac{3}{1-\sqrt{2}}$

$\frac{3}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$ MULTIPLY BY THE CONJUGATE

$= \frac{3(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}$ SIMPLIFY

$= \frac{3+3\sqrt{2}}{1-2}$

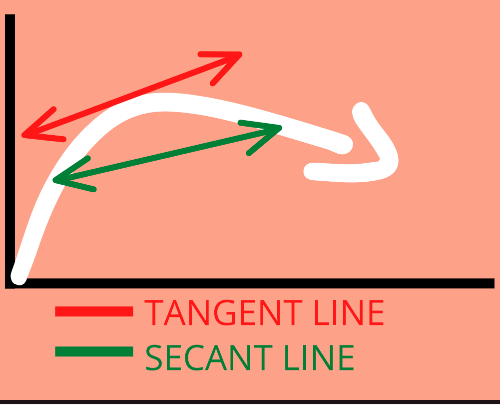
$= \frac{3+3\sqrt{2}}{-1}$

$= -3-3\sqrt{2}$

SLOPE OF A TANGENT

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\Delta y}{\Delta x} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

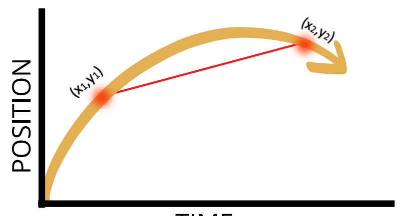


RATE OF CHANGE

Average Rate of Change

(Average Velocity)

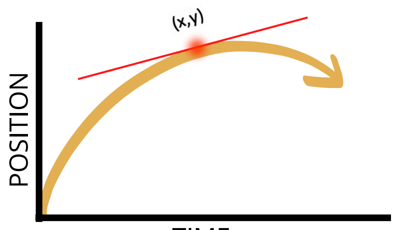
$$\text{ARoC} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Instantaneous Rate of Change

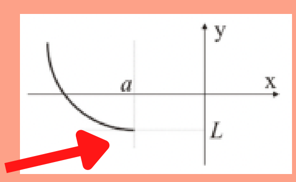
(Instantaneous Velocity)

$$\text{IRoC} = \frac{\Delta y}{\Delta x} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$



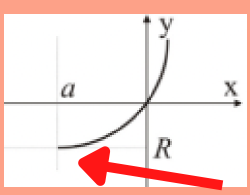
LIMIT OF A FUNCTION

$\lim_{x\to a^-} f(x)$



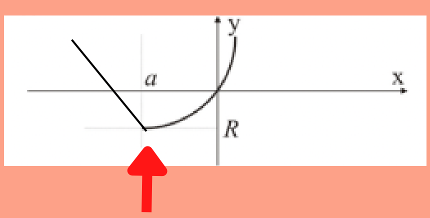
Approaching from the left

$\lim_{x\to a^+} f(x)$



Approaching from the right

$\lim_{x\to a} f(x)$



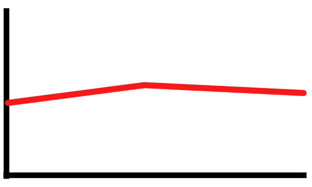
value exactly at that spot

PROPERTIES OF LIMITS

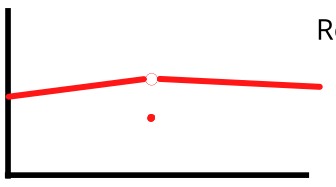
We assume that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. Then:

1. $\lim_{x\to a} k = k$
2. $\lim_{x\to a} x = a$
3. $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
4. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$
5. $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \quad \lim_{x\to a} g(x) \neq 0$
6. $\lim_{x\to a}[f(x)g(x)] = [\lim_{x\to a} f(x)][\lim_{x\to a} g(x)]$
7. $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$
8. If $P(x)$ is a polynomial function, then $\lim_{x\to a} P(x) = P(a)$

CONTINUITY

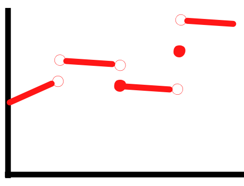


The function is continuous if the left and right limits are equal. And is not a "Point Discontinuity"



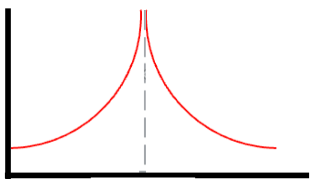
Removable/Point discontinuity

only if: 1. $\lim_{x\to a} f(x)$ exists
2. $f(a)$ Does Not Exists



Jump discontinuity

$\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$



Infinite Discontinuity

If either/both of the left-sided or right-sided limits approach $\pm\infty$